Calculations were done for large H and showed that the features of cloud evolution discussed above (Figs. 1-3) also apply when the cloud falls in an unbounded space; the curves shown in Figs. 4, 5 apply quantitatively to this case.

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TWO-PHASE THREE-COMPONENT FILTRATION WHEN OIL IS DISPLACED BY A SOLUTION OF AN ACTIVE ADDITIVE

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Among the new methods for increasing the output of oil from rock strata, an important place is occupied by processes in which the oil is displaced by solutions of active additives: carbon dioxide gas or surface-active substances. Self-similar solutions were obtained earlier for the case of frontal displacement of the oil by dilute solutions of the additives [1, 2]. At high concentrations of a pumped-in solution the transition of the additive from the injection phase to the oil phase leads to an increase in the mobility of the oil and has a substantial effect on the displacement process. In [3] solutions were obtained for the problem of forcing oil out with solutions of any concentrations, on the assumption that the total volume of the phase remained constant when dissolution took place, and we obtained a number of solutions for problems of frontal displacement. In the present study this system of equations is considered in connection with an active additive which can be dissolved in water and oil but does not cause interphase mass exchange between the water and oil components. We investigate the problem of the decomposition of an arbitrary discontinuity, and we obtain self-similar solutions for problems of frontal displacement with arbitrary values of flooding of the stratum and any forms of the distribution function of the additive between the phases. From the solution of the problem of the structure of the discontinuity, we obtain the conditions for stability of the generalized solution. We investigate typical interactions of simple waves and shock waves, and we obtain solutions for problems involving displacement of the oil by a dose of the solution of active additive forced through the stratum by water.

1. Analysis of the Initial System of Equations. In the displacement process the additive is distributed between the water and oil phases. The system of equations of a two-phase three-component filtration consists of the equation of discontinuity for the water component, the oil component, and the active component [5]. When we consider large-scale displacement processes, we disregard the capillary jump in pressure between the phases,

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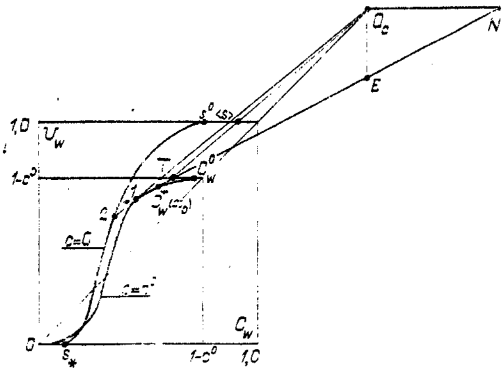

Fig. 1

the diffusion of the additive, and the nonequilibrium distribution of the additive between the phases. We disregard the adsorption of the additive. We assume that in the process of distribution of the additive between the phases the total volume of the mixture remains constant:

$$\rho_{\mathrm{I}} = c\rho_A + (1 - c)\rho_W, \quad \rho_{\mathrm{II}} = \varphi\rho_A + (1 - \varphi)\rho_O,$$

where $\rho_{\mathrm{I}}, \rho_{\mathrm{II}}$ are the densities of the water and oil phases; ρ_A, ρ_W, and ρ_O are the densities of the additive, water, and oil components; c and φ are the volumetric concentrations of the additive in the water and oil phases. Then the function relating the mass concentrations of the additive in the oil and water phases, which exists in the state of thermodynamic-equilibrium distribution of the additive between the phases, can be rewritten in the form of a function connecting the volumetric concentrations — the function giving the distribution of the additive between the phases, $\varphi = \varphi(c)$. The total flow rate of the phases remains constant, and each of the equations of discontinuity for the components is a consequence of the other two. The system of two-phase three-component filtration takes the form [4]

$$\frac{\partial}{\partial t}\{(1-c)s\} + \frac{\partial}{\partial x}\{(1-c)F\} = 0,$$
$$\frac{\partial}{\partial t}\{cs + \varphi(1-s)\} + \frac{\partial}{\partial x}\{cF + \varphi(1-F)\} = 0, \tag{1.1}$$

where x is the ratio of the pore volume, calculated over the stratum from a injection borehole (gallery) to the volume of the dose; t is the ratio of the volume of the pumped-through liquid to the volume of the dose; s(x, t) is the saturation of the pore space with the water phase; F(s, c) is the Buckley—Leverett function, equal to the fraction of the water phase in the flow. In Fig. 1 the curve c = 0 is the graph of the function F = F(s, 0).

Now we pass in the system (1.1) to the unknowns $C_W = (1 - c)s$ (the volumetric concentration of water in the flow) and $U_W = (1 - c)F$ (the volume fraction of water in the total rate of filtration of both phases):

$$\frac{\partial C_W}{\partial t} + \frac{\partial U_W}{\partial x} = 0, \quad \frac{\partial}{\partial t}(\alpha C_W + \varphi) + \frac{\partial}{\partial x}(\alpha U_W + \varphi) = 0, \tag{1.2}$$
$$\alpha(c) = (c - \varphi)(1 - c)^{-1}, \quad \varphi'(c) > 0, \quad \varphi(0) = 0.$$

The curves F = F(s, c), c = const, can be reconstructed into the curves $U_W = U_W(C_W, c)$, c = constant, by compression by a factor of 1 − c along the C_W and U_W axes. In Fig. 1 the curve $c = c^0$ is the graph of the function $U_W = U_W(C_W, c^0)$. By virtue of the relation $U_W = U_W(C_W, c)$, we shall regard both (C_W, c) and (C_W, U_W) as unknowns for the system (1.2).

The system (1.2) is a hyperbolic system of two quasilinear equations. We shall write it in Riemann invariants [6].

To the eigenvalues of the hyperbolic system

$$\xi_1 = \partial U_W/\partial C_W, \quad \xi_2 = (U_W + \varphi'/\alpha')(C_W + \varphi'/\alpha')^{-1}$$

there correspond the two families of simple waves

$$\frac{dU_W}{dC_W} = \frac{\partial U_W}{\partial C_W} = \xi_1, \quad \frac{dU_W}{dC_W} = \frac{U_W + \varphi'/\alpha'}{C_W + \varphi'/\alpha'} = \xi_2 \tag{1.3}$$

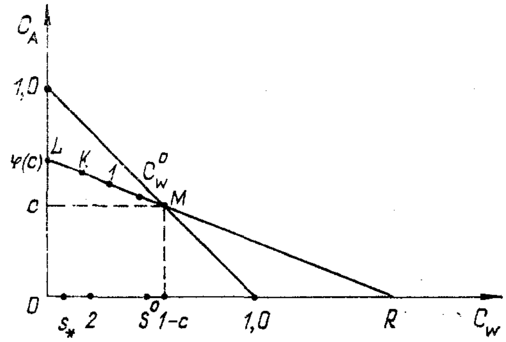

Fig. 2

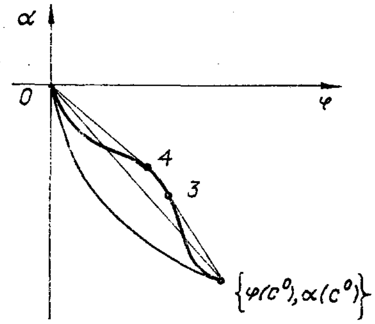

Fig. 3

and the two families of characteristics

$$dx/dt = \xi_1,\ dI(C_W,\ U_W)/dt = 0;\ dx/dt = \xi_2,\ dc(C_W,\ U_W)/dt = 0. \tag{1.4}$$

The invariant which is constant along the C_W characteristics (the first equation in (1.4)) is the arbitrary function $I = I(C_W, U_W)$, constant along the trajectories of the vector field, which is given by the second equation in (1.3), and varying monotonically to trajectory. The invariant which is constant along the c characteristics (the second equation in (1.4)) is the concentration c of the additive in the water phase. When we transform the hodograph $(x, t) \to (C_W(x, t), U_W(x, t))$, the C_W characteristics are carried into simple c waves (the second equation in (1.3)), and the c characteristics into simple C_W waves (the first equation in (1.3)). The characteristic rates of the system are ξ_1 and ξ_2. To a simple C_W wave in the phase plane of the system (C_W, U_W) there corresponds a curve c = const. To a simple c wave in the plane (C_W, U_W) there corresponds a trajectory such that the tangent to it at each point $\{C_W, U_W(C_W, c)\}$ passes through the point (R', R'), $R' = -\varphi'/\alpha'$.

In the plane (C_W, C_A) (where C_A is the volumetric concentration of the additive in the two phases), we shall represent by dots the volumetric compositions of the water phase, $M(1 - c, c)$, the oil phase, $L(0, \varphi(c))$, and both phases, $K(C_W, C_W)$, respectively (Fig. 2). From the formulas $C_W = (1 - c)s$, $C_A = cs + \varphi(1 - s)$ it follows that the points K, L, and M lie on the same straight line and the saturations of the phases are determined by the spring rule, $s = KL/LM$, $1 - s = KM/LM$. If the function $\varphi = \varphi(c)$, is known, then the triangle $\{C_W \geqslant 0,\ C_A \geqslant 0,\ C_W + C_A \leqslant 1\}$ is covered by the straight lines LM, which we shall call nodes. In Fig. 2 we illustrate the case of an additive which is preferentially soluble in oil $\varphi > c$, with nodes inclined to the C_W axis at an obtuse angle $\arctan\alpha$. If the additive is soluble in water better than in oil, the tangent of the angle of inclination of the nodes is $\alpha > 0$.

2. Discontinuous Solutions. The hyperbolic system of conservation laws (1.2) admits of discontinuous solutions. The Hugoniot conditions for the balance of water mass and the balance of additive mass at the discontinuity have the form [6]

$$[C_W]V = [U_W],\ \ [\alpha C_W + \varphi]V = [\alpha U_W + \varphi],$$

where [A] is the jump of the quantity A, equal to the difference between the values before the discontinuity, A^+, and after the discontinuity, A^-; V is the velocity of the discontinuity.

For $[c] \neq 0$, the Hugoniot conditions are transformed to

$$V = \{U_W^{\pm} + [\varphi]/[\alpha]\}\{C_W^{\pm} + [\varphi]/[\alpha]\}^{-1}, \tag{2.1}$$

and for $[c] = 0$ to

$$V = (U_W^+ - U_W^-)(C_W^+ - C_W^-)^{-1}. \tag{2.2}$$

The condition (2.1) means that the points (C_W^-, U_W^-) and (C_W^+, U_W^+) lie in the same straight line, which passes through the point $\{-[\varphi]/[\alpha],\ -[\varphi]/[\alpha]\}$. The inclination of the straight line is equal to the velocity of the discontinuity. Suppose that two nodes corresponding to the values c^+ and c^- in the plane (C_W, C_A) intersect at a point with coordinates (R, T). Then $\varphi^{\pm} - T = -R\alpha^{\pm}$, $R = -[\varphi]/[\alpha]$ is the abscissa of the point of intersection of the nodes. Letting [c] approach zero, we find that $R' = -\varphi'/\alpha'$ is the abscissa of the instantaneous center of rotation of the node.

At those points of the phase plane (C_W, U_W) at which $dR'/dc = 0$ the equation $d\xi_2/d\xi = 0$ is satisfied. Therefore the hyperbolic system (1.2) is not convex (truly nonlinear) [7]. In the case $dR'/dc \equiv 0$ we have $d\xi_2/d\xi \equiv 0$, and the simple c waves go over into contact c discontinuities. We have $R = R' = \text{const}$, $\varphi = -R\alpha$. The Riemann invariant which is constant along the C_W characteristics can be expressed explicitly as $I = (R - U_W)(R - C_W)^{-1}$. The conditions at the c discontinuity (2.1) in this case take on the form $V = I^{\pm}$. All the nodes in the plane (C_W, C_A) intersect at one point, which has the coordinates (R, 0). The variation of the volumetric concentration of the additive in the oil as a function of the volumetric concentration of the additive in the water has the Langmuir form $\varphi = Rc(R - 1 + c)^{-1}$ The relation between the mass concentrations also has a Langmuir form.

As the criterion for stability at the discontinuity in the hyperbolic system (1.2) we take the following two conditions:

1) The total number of characteristics in the zone before the jump with a velocity not exceeding the velocity of the discontinuity, and in the zone after the jump with a velocity not less than the velocity of the discontinuity, is three;

2) in the interval between c^- and c^+ the sign of the expression

$$\{\varphi(\alpha) - \varphi(c^-) - (\alpha - \alpha(c^-))\,[\varphi]/[\alpha]\}$$

coincides with the sign of the difference ($c^+ - c^-$).

Condition 1 coincides with the generalization of the condition for stability of the discontinuity in the Lax form for the case of nonconvex hyperbolic systems; the characteristics whose velocity is equal to the velocity of the discontinuity are taken to be the ones coming into the discontinuity [7]. This condition ensures the unambiguous solvability of the linearized problem of interaction between a discontinuity and a small perturbation; the discontinuity is stable with respect to interaction with a small perturbation.

The nonlinearized problem of the interaction of the discontinuity with a small perturbation is investigated by constructing a transformation of the hodograph, using the notation of the system (1.2) in Riemann invariants. In the case when the Lax condition 1 is satisfied but condition 2 is not, there is a reversal of the front of the small perturbation until this perturbation reaches the discontinuity. This gives rise to a complex configuration, which does not converge to the initial discontinuity as time goes on. If conditions 1 and 2 are satisfied, then when we solve the nonlinearized problem of the interaction of the discontinuity with a small perturbation, the characteristics bring to the discontinuity a number of relations sufficient, together with the Hugoniot conditions, to determine unambiguously the quantities c^-, c^+, C_W^-, C_W^+ and V; the discontinuity is stable.

From conditions 1 and 2 we can derive the well-known generalization of O. A. Oleinik's stability condition for the case of nonconvex hyperbolic systems: The discontinuity is stable if the points (C_W^-, U_W^-) and (C_W^+, U_W^+) and can be connected by a continuous curve $(C_W(\eta)$, $U_W(\eta))$, on which the Hugoniot conditions are satisfied for the jump $(C_W^-, U_W^-) \to (C_W(\eta), U_W(\eta))$, where $\forall\eta V(\eta) \geqslant V$ [7]. In comparison with the Lax condition 1, the Oleinik condition adds the requirement that in the plane (α, φ) the parametrically specific curve $\{\alpha(c), \varphi(c)\}$ does not intersect a segment connecting points before and after the discontinuity (Fig. 3).

The Hugoniot conditions and the stability conditions 1 and 2 are conditions for the existence of a structure when we introduce into the system (1.1) a capillary jump in the pressure between the phases and the kinetics of the process of dissolution of the additive in the oil phase:

$$\begin{gathered}\frac{\partial}{\partial t}\{(1-c)s\} + \frac{\partial}{\partial x}\{(1-c)F\} = \hbar A^0 \frac{\partial}{\partial x}\left\{(1-c)A(s,c)\frac{\partial s}{\partial x}\right\},\\ \frac{\partial}{\partial t}\{cs + \varphi'(1-s)\} + \frac{\partial}{\partial x}\{cF + \varphi'(1-F)\} = \hbar A^0 \frac{\partial}{\partial x}\left\{(c-\varphi')A(s,c)\frac{\partial s}{\partial x}\right\},\\ \partial\varphi'/\partial t = [\varphi(c) - \varphi']/\hbar\tau, \quad A(s,c) > 0, \quad A^0 = \text{const}.\end{gathered} \tag{2.3}$$

Here φ' is the running value of the concentration of the additive in the oil phase; τ is the characteristic time for the establishment of a thermodynamic equilibrium distribution of the additive in the two phases; $\hbar$ is a small parameter. The relation $\varphi' = \varphi(c)$, corresponding to the equilibrium distribution of the additive between the phases, is replaced by the equation of linear kinetics of the dissolution of the additive in the oil phase.

The discontinuity $(s^-, c^-) \to (s^+, c^+)$ in the solution of the system (1.1) admits of the structure (2.3) if it can be obtained as the limit of the solutions of the system (2.3) as $\hbar \to 0$. In a neighborhood of the discontinuity for small values of $\hbar$ we look for a solution in the form of a traveling wave $s(\omega)$, $c(\omega)$, $\varphi'(\omega)$ where $\omega = (x - Vt)/\hbar$. After substituting the form of the solution into the system (2.3), we obtain a system of three ordinary differential equations. We integrate the first two equations with respect to ω, to within constants:

$$\begin{gathered} A^0(1-c)A(s, c)ds/d\omega = (1-c)F - V(1-c)s + \text{const}, \\ A^0(c-\varphi')A(s, c)ds/d\omega = cF + \varphi'(1-F) - V\{cs + \varphi'(1-s)\} \\ + \text{const}, \\ d\varphi'/d\omega = (\varphi' - \varphi(\omega))/V\tau. \end{gathered} \tag{2.4}$$

If $\hbar \to 0$, then $\omega \to \infty$ when $x - Vt > 0$ and $\omega \to -\infty$ when $x - Vt < 0$. Then the condition for the existence of the structure (2.3) is the condition for the existence of a continuous solution of the following boundary-value problem for the system (2.4):

$$s(\pm\infty) = s^\pm,\ c(\pm\infty) = c^\pm,\ \varphi'(\pm\infty) = \varphi(c^\pm). \tag{2.5}$$

If the boundary-value problem (2.5) is solvable, then the points $\{s^-, c^-, \varphi(c^-)\}$ and $\{s^+, c^+, \varphi(c^+)\}$ are singular for the vector field (2.4). From this we obtain the Hugoniot conditions for the system (1.1). If we subtract from the first equation of (2.4) multiplied by $c - \varphi'$ the second equation multiplied by 1 − c, we obtain $\{\varphi' - \varphi(c^-)\}\{\alpha(c) - \alpha(c^-)\}^{-1} = R$. This means that the points $\{\varphi'(\omega), \alpha(\omega)\}$ lie on the segment connecting the points $\{\varphi(c^-), \alpha(c^-)\}$ and $\{\varphi(c^+), \alpha(c^+)\}$. We can therefore express c in terms of φ' and substitute into the third equation of (2.4)

$$d\varphi'/d\omega = \{\varphi' - \varphi(c(\varphi'))\}/V\tau. \tag{2.6}$$

The boundary-value problem $\varphi'(\pm\infty) = \varphi(c^\pm)$ is solvable if and only if the sign of the difference $(c^+ - c^-)$ coincides with the sign of the right side of Eq. (2.6) [6]. When $c^+ < c^-$, this means that the line segment connecting the points $\{\varphi(c^-), \alpha(c^-)\}$ and $\{\varphi(c^+), \alpha(c^+)\}$ lies above the curve $\{\varphi(c), \alpha(c)\}$. We have thus obtained condition 2. The physical interpretation of the fact that $\varphi' < \varphi(c)$ is that when $c^+ < c^-$, in a neighborhood of the discontinuity, the equilibrium concentration of the additive in the oil is greater than the running value of the concentration, and consequently the transition of the additive takes place from the displacing phase to the displaced phase. In a neighborhood of the discontinuity the process of redistribution of the additive between the phases is unidirectional.

Theorem. The boundary-value problem (2.5) for the system (2.4) is uniquely solvable if and only if the Hugoniot conditions and stability conditions 1 and 2 are satisfied.

3. Construction of Self-Similar Solutions.

We consider the process of displacement of the oil by the solution of active additive with concentration c^0 from an unexploited stratum with saturation s_* of bound water. The corresponding initial and boundary conditions for the system (1.2) have the form

$$C_W(x, 0) = s_*,\ c(x, 0) = 0,\ C_W(0, t) = C_W^0,\ c(0, t) = c^0. \tag{3.1}$$

Here $C_W^0 = (1 - c^0)s^0(c^0)$; $s^0(c^0)$ is the limiting saturation of the displacing phase when an additive solution with concentration c^0 is used. To the condition for the injection borehole (gallery) corresponds the point C_W^0, lying at the node $c = c^0$ (see Fig. 2). To the condition in the stratum corresponds the point s_*, lying on the straight line $C_A = 0$, and the additive is not present.

The problem (3.1) is the problem of the breakdown of the discontinuity for the system (1.1). It admits of a self-similar solution $C_W(\xi)$, $c(\xi)$, $\xi = x/t$. The self-similar solution of the system (1.1) can conssit of the following elements:

— centered C_W waves; the corresponding segment in the phase plane of the system (C_W, U_W) will be denoted by C_W;

— centered c waves; the corresponding segment of the motion in the plane (C_W, U_W) will be noted by c;

— C_W discontinuities; the corresponding notation is $(C_W^-, c) - J \to (C_W^+, c)$;

— c discontinuities; the notation is $(C_W^-, c^-) - Jc \to (C_W^+, c^+)$;

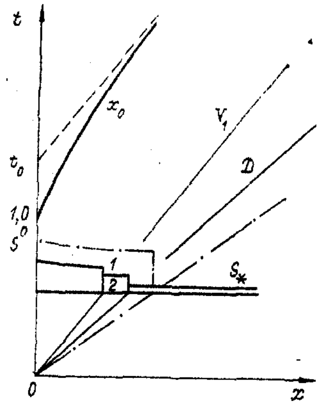

Fig. 4

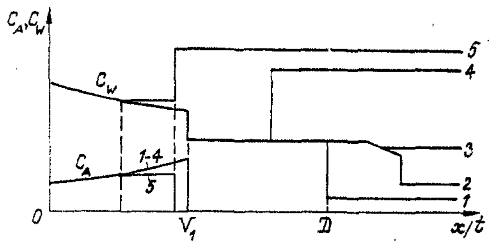

Fig. 5

— zones of rest, i.e., of constant C_W and c; the notation is P.

The self-similar substitution reduces the condition (3.1) to the form

$$C_W(0) = C_W^0,\ c(0) = c^0,\ C_W(\infty) = s_*,\ c(\infty) = 0. \tag{3.2}$$

The solution of the problem (3.2) consists in finding a path $(C_W(\xi), U_W(\xi))$ in the plane (C_W, U_W) that will connect the points (C_W^0, c^0) and $(s_*, 0)$ and may consist of the five elements listed above.

We consider first the case of a distribution function for the additive between the phases $\varphi = \varphi(c)$ such that in the interval between the values $c = 0$ and $c = c^0$ in the plane (φ, α) the segment connecting the points $(0, 0)$ and $\{\varphi(c^0), \alpha(c^0)\}$ lies no lower than the curve $\{\varphi(c), \alpha(c)\}$. From the point $O_c(R, R)$, where $R = -\varphi(c^0)/\alpha(c^0)$, we draw the tangent $O_c - 1 - 2$ to the curve $c = c^0$ (see Fig. 1).

The path corresponding to the solution of the problem (3.2) consists of motion in a simple C_W wave from the point C_W^0 to the point 1, a c jump from the point 1 to the point 2 with velocity V_1, a zone of rest at the point 2, and a C_W jump from the point 2 to the point s_* with velocity D:

$$(C_W^0, c^0) - C_W - (C_W^1, c^0) - Jc \to (C_W^2, 0) - P - J \to (s_*, 0). \tag{3.3}$$

Both jumps appearing in the solution are stable. Their velocities are found from the conditions at the discontinuities. Analogously [9] it can be shown that at the c discontinuity the Jouguet conditions $V_1 = \partial U_W(C_W^1, c^0)/\partial C_W$ are satisfied.

The self-similar solution has the following form:

$$\begin{gathered} x/t = \partial U_W/\partial C_W,\ c = c^0,\ 0 < x/t < V_1 = \partial U_W(C_W^1, c^0)/\partial C_W \\ = (U_W^1 - R)(C_W^1 - R)^{-1}, \\ C_W = C_W^2,\ c = 0,\ V_1 = (U_W^2 - R)(C_W^2 - R)^{-1} < x/t < D, \\ C_W = s_*,\ c = 0,\ D = U_W^2(C_W^2 - s_*)^{-1} < x/t < \infty. \end{gathered}$$

In Fig. 4, for t < 1, we show the profiles for the distribution in the stratum of the concentrations of the water and the additive when the oil is displaced by a solution of active additive (solid curve) and by water (dashed curve). The structure of the displacement zone is the following: Beyond the zone of the displaced oil there is a water-oil swell with no additive, and after that comes a zone of dissolution of the additive, whose concentration in the water phase is equal to c^0. The water content of the output in the zone of the swell is equal to U_W^2 (point 2 in Figs. 1-3). The occurrence of a water-oil swell is due to the presence of water in the stratum before the start of the exploitation. The solubility of the additive in the oil increases the separation of the displacement front x = Dt from the front of concentration of the pumped-in additive $x = V_1t$. With increasing $\varphi(c)$ the velocity V_1 decreases, D increases, and the zone of the water-oil swell expands.

At the front $x = V_1t$ there is a complete jump in concentration. The mapping point in Figs. 1 and 2 moves from the point C_W^0 at the borehole to the point 1 at the front. The concentration of the additive increases from C_A^0 to C_A^1 (see Fig. 2). The increase in the additive concentration in the stratum from the injection borehole is an interesting feature related to the preferential dissolution of the additive in oil. If the additive dissolves better in water, then as we move along the node $c = c^{\circ}$ from the point C_W° to the point 1, the value of C_A decreases. From Fig. 3 it can be seen that in comparison with the usual flooding, the use of an additive increases the period of water-free operation and reduces the volumetric concentration of water. Because part of the additive goes over into the displaced phase and the value of the residual oil saturation decreases when the oil is displaced by a solution of active additive $\{1 - s^0(c^0)\}\ \{1 - \varphi(c^0)\}$, in comparison with the usual flooding $1 - s^{\circ}$, there is an increase in the degree of displacement in the last stage of the working.

Figure 5 shows the profiles of the distribution of the values of C_W and C_A for various values of the initial flooding of the stratum, s(x, 0). Curve 1 corresponds to the values $s_* < s(x, 0) < s'$, where s' is the point of intersection of the tangent to the curve c = 0 at the point 2 with the curve c = 0. Curve 2 is constructed for the values $s' < s(x, 0) < s''$, where s" is the point of inflection of the curve c = 0. The C_W jump at the point s(x, 0) is preceded by motion in the centered C_W wave along the curve c = 0. The profile of the quantity C_W during the initial period of flooding coincides with the profile of the water saturation in the usual flooding. The same thing happens in the case of $s'' < s(x, 0) < C_W^2$ (curve 3). Curve 4 corresponds to the complete displacement of the oil from a heavily flooded stratum with an initial water saturation of $s(x, 0) > C_W^2$. Part of the additive from the displacing phase goes into the oil phase, which increases its saturation and mobility. For small values of c°, when the straight line passing through the point O_c and s(x, 0) intersects the curve $c = c^{\circ}$, there arises the situation of complete displacement corresponding to curve 5. Beyond the unperturbed zone of the displaced oil and water there is an oil plateau containing the additive. Separation of the additive front from the displacement front does not take place.

Let us consider the case of an arbitrary curve $\{\varphi(c), \alpha(c)\}$. We construct a convex envelope of the curve in the interval $(0, c^{\circ})$ — the graph of the minimal convex function $\varphi = \varphi(\alpha)$, whose points lie no higher than the curve (see Fig. 3). The envelope consists of segments of arcs of the curve and tangents to the curve. From Fig. 3 it can be seen that $\varphi'(c_3)/\alpha'(c_3) = \{\varphi(c_3) - \varphi(c^0)\}/\{\alpha(c_3) - \alpha(c^0)\} = -R_3,\ \varphi'(c_4)/\alpha'(c_4) = \varphi(c_4)/\alpha(c_4) = -R_4$. In the self-similar solution of the problem of frontal displacement (3.1) the transition from the zone $c = c^{\circ}$ into the swell zone c = 0 is accomplished not by a full c jump but by a sequence of c jumps and centered c waves. We have c waves corresponding to the arc segments of the curve $\{\varphi(c), \alpha(c)\}$ in the envelope, and c jumps corresponding to the tangents. Corresponding to the solution of the problem (3.1) for the case of the curve shown in Fig. 3, in the plane (C_W, U_W), we have the path

$$(C_W^0, c^0) - C_W - (C_W^1, c^0) - Jc \to (C_W^3, c_3) - c - (C_W^4, c_4) - Jc \to (C_W^2, 0) - P - J \to (s_*, 0).$$

The velocities of the first and second c jumps in the solutions are found from the Hugoniot condition on the discontinuity and the Jouguet condition

$$V_3 = \frac{R_3 - U_W(C_W^1, c^0)}{R_3 - C_W^1} = \frac{\partial U_W(C_W^1, c^0)}{\partial C_W},\ V_4 = \frac{R_4 - U_W(C_W^4, c_4)}{R_4 - C_W^4}.$$

Between the zone of pumped-in solution $c = c^{\circ}$ and the swell c = 0 we find a zone in which the value of c varies continuously from c_3 to c_4.

It should be noted that for the curve $\{\varphi(c), \alpha(c)\}$ under consideration, the problem (3.1) admits of a self-similar solution (3.3) in which the complete jump in the concentration is stable in Lax's sense but unstable in Oleinik's sense. It can be proved that the problem of the breakdown of an arbitrary discontinuity for a hyperbolic system (1.1) has a unique generalized self-similar solution at whose discontinuities the stability conditions 1 and 2 are satisfied.

4. Displacement of Oil by a Dose of Solution of Active Additive. Because the additives are so expensive, solutions of them are pumped in the form of finite volumes (doses) moved through the stratum by the water. The corresponding initial and boundary conditions for the system (1.2) have the form

$$C_W(x, 0) = s_*,\ c(x, 0) = 0, \quad C_W(0, t) = \begin{cases} C_W^0, & t < 1, \\ 1, & t > 1, \end{cases} \tag{4.1}$$

$$c(0, t) = \begin{cases} c^0, & t < 1, \\ 0, & t > 1. \end{cases} \tag{4.1}$$

Up to time t = 1 it is the solution of additive that is pumped into the stratum, but after that time it is the water. At t < 1 the solution of the problem (4.1) coincides with the self-similar solution of the problem of frontal displacement of the oil by the additive solution. At time t = 1 there is a breakdown of the discontinuity of the boundary condition $C_W^- = 1$, $c^- = 0$, $C_W^+ = C_W^0$, $c^+ = c^0$, and we begin to observe the interaction of the breakdown configuration with the C_W wave of the self-similar solution.

We consider the contact case of the Langmuir function of the distribution of the additive between the phases, $\varphi = -R\alpha$. To the configuration of the breakdown in the discontinuity of the boundary condition there corresponds the path in the plane (C_W, U_W) $(1, 0) - J \to (s_5, 0) - P - Jc \to (C_W^0, c^0)$. According to the condition (2.1), the point s_5 lies at the intersection of the straight lines $O_c - (C_W^0, c^0)$ and $U_W = 1$. The resulting contact c discontinuity if $x_0(t)$ — three doses — is propagated along the c characteristic. On this there is a complete jump in the concentration, $c^-(x_0) = 0$, $c^+(x_0) = c^0$. Since in the region of the centered wave the inequality $x/t = \partial U_W/\partial C_W < (R - U_W)(R - C_W)^{-1} = dx_0/dt$ is satisfied, it follows that all the C_W characteristics, which are rays of the centered C_W wave, intersect the curve of discontinuity $x = x_0(t)$. They bring the values of the invariant I^+ to the curve of discontinuity x_0. From this we have

$$x_0/t = \partial U_W(C_W^+(x_0), c^0)/\partial C_W, \; dx_0/dt = I^{\pm}(x_0). \tag{4.2}$$

As the discontinuity in the centered wave of the quantity $C_W^+(x_0)$ decreases from C_W^0 to C_W^1, the velocity of the discontinuity increases to the value V_1 (see Fig. 1).

We integrate the second equation in (1.2) over the region of the plane (x, t) bounded by the contour $(0, 0) \to (0, 1) \to (x_0, t) \to (0, 0)$ (see Fig. 4). According to Green's formula, the integral along this contour of the differential form $\Theta_A = (\alpha U_W + \varphi)dt - (\alpha C_W + \varphi)dx$ is equal to zero. The form Θ_A has the meaning of the volumetric flow of the additive. The integral of Θ_A over the segment $(0, 0) \to (0, 1)$ is equal to $\alpha(c^0)(1 - c^0) + \varphi(c^0) = c^0$. Since $c^-(x_0) = 0$, the integral of Θ_A along the curve x_0 is equal to zero. The physical meaning of this fact is that there is no return flow of the additive through the curve of the contact discontinuity. Therefore the integral of the form Θ_A over the segment $x_0(t)$ is independent of time and is the first integral of the motion of $x = x_0(t)$ [10]:

$$R - 1 + c^0 = \Delta(C_W^+(x_0), c^0)\, t, \; \Delta(C_W, c) = R - U_W(C_W, c) - (R - C_W)\,\partial U_W/\partial C_W. \tag{4.3}$$

From the system of two transcendental equations consisting of the first equations of (4.2) and (4.3) for any instant of time t we can unambiguously determine the values of $C_W^+(x_0)$ and x_0. From the conditions at the discontinuity (the second equation in (4.2)) it is possible to determine $C_W^-(x_0)$.

The system (4.2), (4.3) can be solved by a geometric construction in the plane (C_W, U_W) [11]. We draw the tangent to the curve $c = c^0$ at the point $C_W^+(x_0)$ until it intersects the straight lines $C_W = R$ and $U_W = R$ at the points E and N, respectively (see Fig. 1). Then by virtue of (4.3), we have

$$O_cE = R - U_W(C_W^+(x_0), c^0) - (R - C_W^+(x_0))\,\partial U_W/\partial C_W = (R - 1 + c^0)/t,$$
$$O_cN = O_cE(\partial U_W/\partial C_W)^{-1} = (R - 1 + c^0)/x_0.$$

To find the position of the back end of the dose at time t we must lay off a segment O_cE and draw a tangent to the curve $c = c^0$. The point $C_W^-(x_0)$ lies at the intersection of the straight line $O_c - C_W^+(x_0)$ and the curve c = 0.

In the zone of forward-pushing water c = 0, and the problem (4.1) for the system (1.2) reduces to the mixed problem $C_W(0, t) = 1$, $C_W(x_0, t) = C_W^-(x_0(t))$ for the first equation of (1.2). The values of $C_W^-(x_0)$ are carried into the zone of forward-pushing water $0 < x < x_0$ along the C_W characteristics. In Fig. 1 $C_W^-(x_0) > s^0$, the inclination of the C_W characteristics is zero, the oil phase is motionless, and complete flooding of the output takes place at the moment when the back end of the dose reaches the extraction gallery.

As $t \to \infty$, we have $C_W^+(x_0) \to C_W^1$, $dx_0/dt \to V_1$, the volume of the dose $\Omega(t) = V_1 t - x_0(t)$ increases. We integrate the second equation of (1.2) over the region of the plane (x, t) bounded by the contour $(0, 0) \to (0, 1) \to (x_0, t) \to (V_1 t, t) \to (0, 0)$. Since the integrals of the form Θ_A along the curves of the contact discontinuities $x = x_0(t)$ and $x = V_1 t$ are equal to zero, we have

$$c^0 = \int\limits_{x_0(t)}^{V_1 t} \{\alpha(c^0) C_W(x, t) + \varphi(c^0)\} dx.$$

The meaning of the expression obtained above is the balance of the additive in the dose. If we let $t \to \infty$ in this expression, we obtain $\Omega(\infty) = (R - 1 + c^0)(R - C_W^1)^{-1}$. The volume of the dose stabilizes as time goes on. Therefore in the case of plane-parallel displacement, the thickness of the dose stabilizes with time, whereas in the case of radial displacement it decreases asymptotically to zero. From time t = 1 the volume of the dose increases by a factor of $t_0 = (R - 1 + c^0)(R - U_W^1)^{-1}$. The back end of the dose has an inclined asymptote $x = V_1(t - t_0)$ (see Fig. 4).

In order to find the average water saturation $\langle s \rangle$ in the stratum at the moment t_W of complete flooding (when the back end of the dose approaches the extraction gallery), we integrate the first equation in (1.2) over the region bounded by the contour $(0, 0) \to (0, t_W) \to (x_0(t_W), t_W) \to (0, 0)$:

$$\langle s \rangle = C_W^+(x_0) + (1 - U_W(C_W^+(x_0), c^0))(\partial U_W/\partial C_W)^{-1} - c^0/x_0.$$

The quantity $\langle s \rangle$ is found as the point of intersection of the straight lines $U_W = 1$ and $O_c T$, where T is the point of intersection of the tangent NE and the straight line $U_W = 1 - c^0$ (see Fig. 1). With increasing volume of the dose (concentration of the additive c^0), the point N (the curve $c = c^0$) is displaced to the right, the value of $\langle s \rangle$ increases, and the oil output increases.

If $dR'/dc < 0$, the back end of the dose passes through the zone of the C_W wave of the self-similar solution and will be propagated in the zone of the centered c wave. The velocity of the back end increases to the velocity of the front of the dose as $t \to \infty$. The additive concentration in the dose will decrease from some value at the back end to zero at the front. When $dR'/dc > 0$, there will be an interaction between the configuration of the self-similar solution and the centered c wave of the configuration of the breakdown of the discontinuity of the boundary condition at time t = 1. When the characteristic $c = c^0$ reaches the front of the dose, the zone $c = c^0$ disappears. The concentration of the additive increases from zero at the back end to some value at the front. In the cases listed as $t \to \infty$, we have $\Omega(t) \sim t^{1/2}$, $C_A \sim t^{-1/2}$; in the case of plane-parallel displacement the thickness of the dose $\sim t^{1/2}$, and in the case of radial displacement it stabilizes.

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THE PERMEABILITY COEFFICIENT OF A POROUS MEDIUM SATURATED WITH A GAS OR LIQUID AFTER A CONFINED EXPLOSION

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Considerable scientific and practical interest attaches to the changes in infiltration properties of a medium produced by a confined explosion. One of the first attempts to determine the permeability coefficient in the region of such an explosion is to be found in [1]. Here we give results from processing experimental data obtained on a porous saturated medium after a confined explosion. The methods of examining the infiltration properties have been described in detail [2, 3]. The pressure difference and the fliud flow rate between different points in the medium were determined in the stationary state. These data are used as initial ones in solving the two-dimensional inverse problem for the permeability coefficient. The method of solving the problem is applied in processing experimental results obtained on a porous saturated medium after a confined explosion.

Experimental Data. A method analogous to that of [2, 3] was used in examining the infiltration properties of the medium after a confined explosion. The experimental explosions were performed in an artificially cemented medium having properties similar to those of real collectors and constituting a mixture of dressed sand, lime flour, and waterglass. The medium was placed in a cylindrical metal vessel of diameter 300 mm and height 350 mm. We used TEN charges of mass 0.4, 0.76, and 1.34 g. Each charge was placed at the middle of the model and was detonated from the center.

A comprehensive study was made of the mechanical effects in the high-porosity medium (m = 25%). The results provided an answer on whether there is any difference in infiltration parameters for monolithic and porous media when acted on by the explosion energy, and what is the difference in these properties if the explosion is performed in a medium in which the pores are filled with air at atmospheric pressure or with a liquid.

Tubes of diameter 3 mm were placed at various distances from the charge between the cavity and the periphery in the models to examine the changes produced by the explosion. The ends of the tubes were perforated and the opposite ends were connected to a measurement system. The tubes were placed in the horizontal or vertical plane of the charge. The model enclosed in the metal cylinder was hermetically sealed by flanges at the ends. Figure 1 shows the disposition of the tubes.

We determine the steady-state flow rate Q_i of air or kerosene and the corresponding pressure difference between a pair of tubes before and after explosion. The infiltration characteristic for the liquid-saturated medium was the ratio $\Gamma = Q_i/\Delta p_i$, where Q_i is the steady-state flow rate and $\Delta p_i = p_{i+1} - p_i$ is the pressure difference between a pair of tubes, while $i = 1, 2, \ldots, N$ represents the tube number. In a gas-saturated medium, Γ is defined by $\Gamma = Q/(p_{i+1}^2 - p_i^2)$.

The parameter change due to the explosion was evaluated from Γ/Γ_0, where Γ_0 is the characteristic before the explosion. Figures 2 and 3 give the results from the experiments.

Inverse Permeability Determination from Experimental Data. The pressure difference Δp_i between tubes is determined not only by the permeability of the medium between them but also by the properties of adjacent regions, as well as by the geometry of the model. It is therefore necessary to solve the inverse problem in order to determine the rational dependence of the permeability coefficient from the results.

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